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SHAPE SEGMENTATION
USING ARC/CHORD PROPERTIES

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ABSTRACT

This report investigates a class of shape segmentation methods in
which, for each arc of the shape's contour, we consider the region bounded
by the arc and its chord; compute a simple geometrical property of this
region; and choose arcs for which this property's value is a local ex-
tremum. The characteristics of this approach are analyzed for several
such properties, and examples of the segmentations obtained in this way
are given. It is concluded that such methods will sometimes yield re-
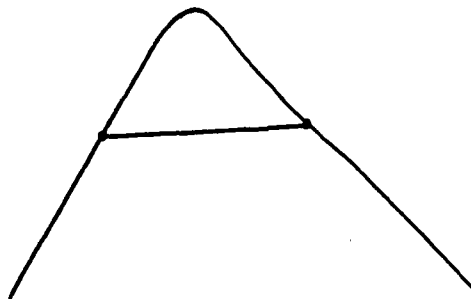sults that are not perceptually plausible.

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1. Introduction

Computer vision tasks frequently involve the segmentation of a shape into parts [1]. Many methods of shape segmentation involve computing some local property of the boundary, such as curvature, and selecting segmentation points based on this measure. One class of measures is obtained by scanning a chord around the boundary, and measuring certain characteristics of the chord and the piece of boundary (the arc) between its endpoints. For example, we could measure the ratio of arc length to chord length:
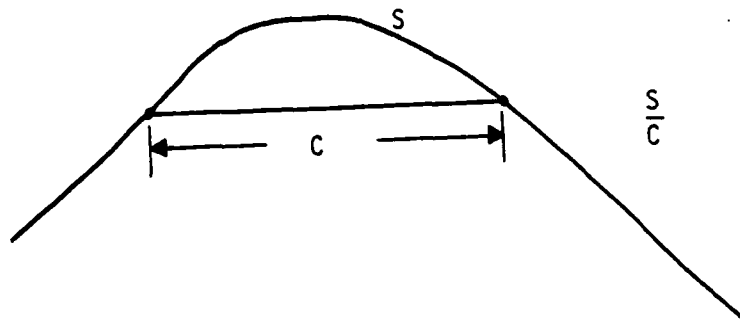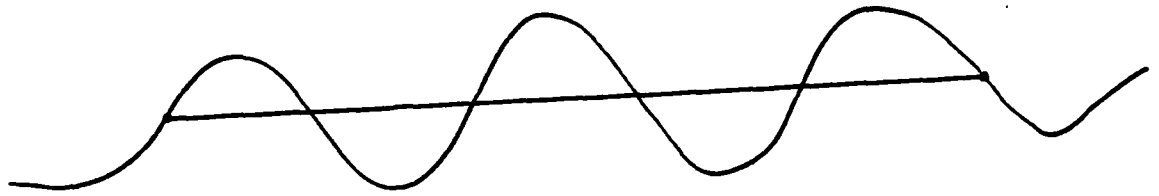

The minimum value of this ratio is obviously 1, and this occurs when the arc is a straight line. $\frac{S}{C}$ takes on higher values in areas where the curve has a sharp angle or is "busy," such as

or

One fault of the measure S/C is its inability to distinguish between a large, simple protrusion, such as a corner, and a series of small oscillations or noise. Other measures, such as the enclosed area divided by the square of the chord length (A/C^2), or the maximum perpendicular distance between the arc and the chord divided by the chord length (D/C), have also been tried.

One property which all implementations of these methods have had in common until now is that although a chord on the shape boundary is defined by two parameters, some auxiliary condition has always been imposed to reduce this to a one parameter problem. For example, one could specify a fixed chord length or a fixed arc length. In this paper we have examined the two parameter problem, in which both chord endpoints are arbitrary. Thus we define a function of two variables, these being the chord endpoints, and look for local minima or maxima of that function. The motivation for this is provided by examples such as the shape below:
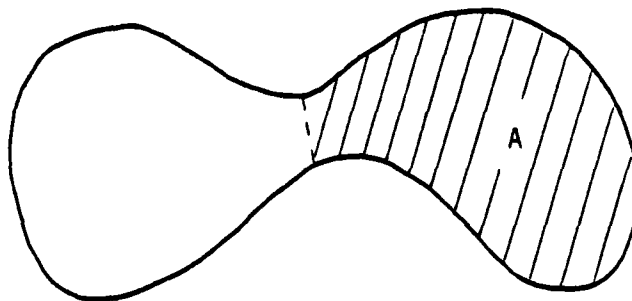
We would expect a measure such as A/c^2 to take on a locally maximum value at the chord shown, which seems to be a reasonable segmentation despite the absence of distinguishing local features, such as corners. One objective of this approach is to provide a segmentation which is based on global rather than local shape properties.

2. Local extrema of A/C^2

Consider a closed curve with an arbitrary starting point, P_0 (Figure 1). We can define a directed chord by selecting two points, P_1 and P_2. Let s_1 be the arc length from P_0 to P_1 measured counterclockwise around the curve, and let s_2 be the arc length from P_0 to P_2 measured in a clockwise direction. Opposite directions are used because of the resulting symmetry in the equations. C is the length of the chord, and A the area enclosed by the chord and the piece of curve (arc) which runs counterclockwise from P_1 to P_2. Area to the right of the directed chord will be considered positive, and area to the left negative.

In the derivations which follow we will require several partial derivatives. These are most easily obtained geometrically. Equations 1 to 4 should be clear from the geometry of Figures 1 and 2. Equation 5 is a bit more difficult to see. When we change s_1 by an amount Δs_1, there are two effects which act together to change θ_1. These are illustrated in Figure 2. If we let k_i be the curvature (defined in the conventional way) at the point i, then when we move a distance Δs_1 along the curve, the curve bends
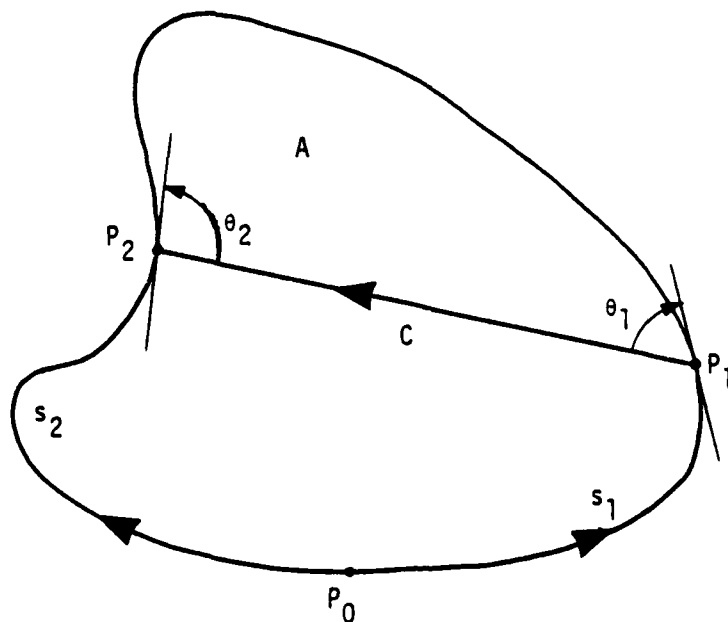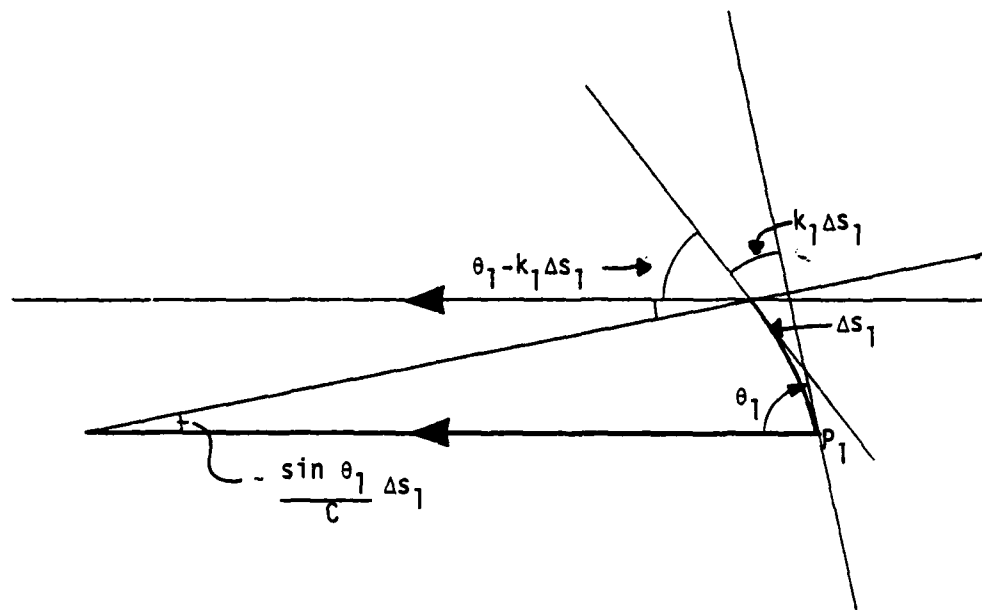

Figure 1.

$$\Delta\theta_1 \cong (\theta_1 - k_1\Delta s_1 + \frac{\sin\theta_1}{C}\Delta s_1) - \theta_1$$

$$\cong (\frac{\sin\theta_1}{C} - k_1) \, \Delta s_1$$

$$\frac{\partial A}{\partial s_i} = -\frac{C}{2}\sin\theta_i \qquad \text{(eq. 1)}$$

$$\frac{\partial C}{\partial s_i} = -\cos\theta_i \qquad \text{(eq. 2)}$$

$$\frac{\partial\theta_2}{\partial s_i} = -\frac{\sin\theta_1}{C} \qquad \text{(eq. 3)}$$

$$\frac{\partial\theta_1}{\partial s_2} = -\frac{\sin\theta_2}{C} \qquad \text{(eq. 4)}$$

$$\frac{\partial\theta_i}{\partial s_i} = \frac{\sin\theta_i}{C} - k_1 \qquad \text{(eq. 5)}$$

Figure 2.

through an angle $K, \Delta s_1$. θ_1, however, is the angle between the tangent to the curve and the chord, so we must also account for the angular change due to our motion relative to point P_2. This change is, to first order, $\Delta s_1 \sin \theta_1/C$. Thus we arrive at equation 5.

We are interested in finding local extrema of the function A/c^2. Specifically, we want to find positive local maxima (A>0) and negative local minima (A<0). Note that there is a symmetry between local maxima and minima. If we reversed our convention of considering area to the right of the directed chord as positive, then the local maxima would become local minima and vice versa. Thus any segment of curve between the endpoints of a chord will meet the criteria for a maximum iff its "minor image" meets the conditions for a minimum (Figure 3).

A necessary condition for the function A/c^2 to have a local extreme value is that the first partial derivatives are zero:

$$\frac{\partial(A/c^2)}{\partial s_i} = \frac{2A}{C^3} \cos \theta_i - \frac{1}{2C} \sin \theta_i = 0 \text{ which implies}$$

$$\tan \theta_i = \frac{4A}{C^2} , \quad i = 1,2 \tag{eq. 6}$$

The standard test for a local minimum or maximum of a function of two variables is

$$\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial y \partial x} \right)^2 \begin{cases} > 0 \text{ then} \dfrac{\partial^2 f}{\partial x^2} > 0 \Rightarrow \text{minimum} \\ \qquad\qquad \dfrac{\partial^2 f}{\partial x^2} < 0 \Rightarrow \text{maximum} \\ < 0 \text{ then neither min nor max} \\ = 0 \text{ undecided.} \end{cases} \tag{eq. 7}$$

+A

local maximum

-A

local minimum
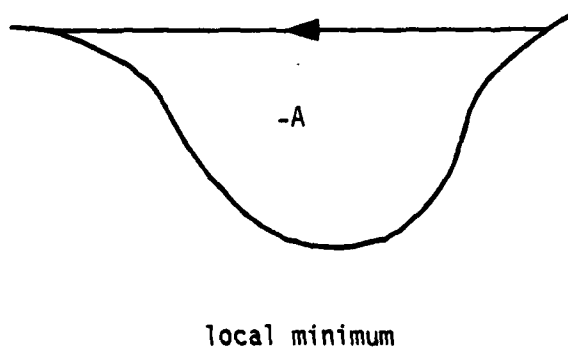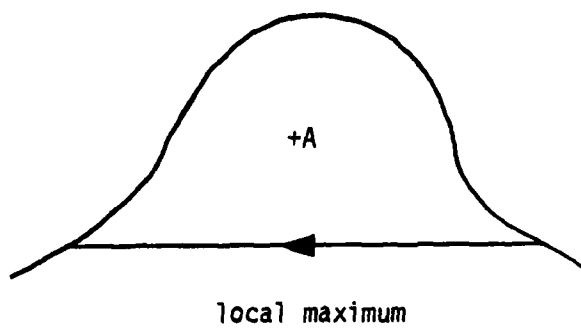
Figure 3.

Evaluating these derivatives at the stationary point:

$$\frac{\partial^2 (A/c^2)}{\partial s_i^2} = \frac{\partial}{\partial s_i} \left[\frac{1}{2C} \left(\frac{4A}{C^2} \cos \theta_i - \sin \theta_i \right) \right] =$$

$$\left(\frac{4A}{C^2} \cos \theta_i - \sin \theta_i \right) \frac{\partial (\frac{1}{2C})}{\partial s_i} + \frac{1}{2C} \left[\frac{\partial}{\partial s_i} \left(\frac{4A}{C^2} \cos \theta_i \right) - \frac{\partial \sin \theta_i}{\partial s_i} \right] =$$

$$\frac{1}{2C} \left[\frac{4A}{C^2} \frac{\partial \cos \theta_i}{\partial s_i} + 4 \cos \theta_i \frac{\partial (A/c^2)}{\partial s_i} - \cos \theta_i \frac{\partial \theta_i}{\partial s_i} \right]$$

$$- \frac{1}{2C} \left[\frac{4A}{C^2} \sin \theta_i \frac{\partial \theta_i}{\partial s_i} + \cos \theta_i \frac{\partial \theta_i}{\partial s_i} \right] = - \frac{1}{2C} \left[\frac{4A}{C^2} \sin \theta_i + \cos \theta_i \right] \frac{\partial \theta_i}{\partial s_i}$$

Substituting (eq. 6) yields:

$$\frac{4A}{C^2} \sin \theta_i + \cos \theta_i = \tan \theta_i \sin \theta_i + \cos \theta_i = \frac{1}{\cos \theta_i}$$

$$\therefore \quad \frac{\partial^2 (A/c^2)}{\partial s_i^2} = \frac{-1}{2C \cos \theta_i} \left(\frac{\sin \theta_i}{C} - k_i \right) \qquad \text{(eq. 8)}$$

Similarly,

$$\frac{\partial^2 (A/c^2)}{\partial s_2 \partial s_1} = \frac{\partial}{\partial s_2} \left[\frac{1}{2C} \left(\frac{4A}{C^2} \cos \theta_1 - \sin \theta_1 \right) \right] =$$

$$\frac{-1}{2C \cos \theta_1} \frac{\partial \theta_1}{\partial s_2} = \frac{\sin \theta_2}{2C^2 \cos \theta_1} \qquad \text{(eq. 9)}$$

Substituting the second partial derivatives into (eq. 7):

$$\left[\frac{-1}{2C \cos \theta_1} \left(\frac{\sin \theta_1}{C} - k_1 \right) \right] \left[\frac{-1}{2C \cos \theta_2} \left(\frac{\sin \theta_2}{C} - k_2 \right) \right] - \left(\frac{\sin \theta_2}{2C^2 \cos \theta_1} \right)^2 =$$

$$\frac{1}{4C^2 \cos \theta_1 \cos \theta_2} \left[\frac{\sin \theta_1 \sin \theta_2}{C^2} + k_1 k_2 - \frac{1}{C}(k_1 \sin \theta_2 + k_2 \sin \theta_1)\right] - \frac{1}{4C^4}\left(\frac{\sin \theta_2}{\cos \theta_1}\right)^2$$

Since we are interested only in the sign of this equation, we can factor out a $\frac{1}{4C^4}$, and note that since (eq. 6) implies $\left(\frac{\sin \theta_2}{\cos \theta_1}\right)^2 = \tan^2 \theta_1 = \frac{\sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2}$, so we obtain

$$\frac{1}{\cos \theta_1 \cos \theta_2} \left[k_1 k_2 - \frac{1}{C}(k_1 \sin \theta_2 + k_2 \sin \theta_1)\right] \qquad \text{(eq. 10)}$$

We note that although (eq. 6) implies $\tan \theta_1 = \tan \theta_2$, these two angles may be either equal, or may differ by π radians. For the moment, let us consider the case where $\theta_1 = \theta_2$, then letting $\theta_1 = \theta_2 = \theta$ and factoring out a $\frac{1}{\cos^2 \theta}$, the above equation becomes simply

$$k_1 k_2 - \frac{\sin \theta}{C} (k_1 + k_2) \qquad \text{(eq. 11)}$$

Consider now the case in which θ_1 and θ_2 differ by π. Suppose $\sin \theta_1$ and $\cos \theta_1$ are > 0 (so sin and cos for θ_2 are < 0). If we let $\theta = \theta_1$, then (eq. 10) becomes

$$\frac{-1}{\cos^2 \theta} \left[k_1 k_2 - \frac{1}{C}(-k_1 \sin \theta + k_2 \sin \theta)\right] \text{ or,}$$

factoring out $\frac{1}{\cos^2 \theta}$,

$$k_1(-k_2) - \frac{\sin \theta}{C} [k_1 + (-k_2)] \qquad \text{(eq. 12)}$$

Thus we get a result similar to (eq. 11) with $-k_2$ in place of $+k_2$. There is a simple intuitive justification for this. Clearly, the conditions for a local max or min defined by (eq. 7) depend only on the value of A and C and the local behavior of the curve at the endpoints of the chord. Consider

the two situations diagramed in Figure 4. Figure 3a, with $\theta_1 = \theta_2$, corresponds to the situation from which we derived (eq. 11). The curve diagramed in Figure 4b has the same local behavior as the curve in 4a. It is therefore not surprising that (eq. 12), which applied to 4b, has the same form as (eq. 11) but with $-k_2$ in place of $+k_2$.

An interesting consequence of this analysis is that the function A/c^2 has no local maximum on a shape whose curvature is everywhere positive.

Suppose a certain chord is assumed to yield a local maximum of A/c^2; We note that for any convex shape $\sin \theta_1$, $\sin \theta_2$, and A must all be ≥ 0. Since at a stationary point $\tan \theta_i = \frac{4A}{c^2}$, we conclude that $\theta_1 = \theta_2$ and both $\sin$ and $\cos$ of θ_1 and θ_2 are positive. Thus (eq. 11) applies here. If $k_1 k_2 - \frac{\sin \theta}{c}(k_1 + k_2) < 0$, then there is neither a local minimum nor maximum.

If $\qquad k_1 k_2 - \frac{\sin \theta}{c}(k_1 + k_2) > 0$, then

$$k_1 k_2 > \frac{\sin \theta}{c} k_1 + \frac{\sin \theta}{c} k_2$$

$$k_1 > \frac{\sin \theta}{c} + \frac{\sin \theta}{c} \frac{k_1}{k_2} > \frac{\sin \theta}{c}$$
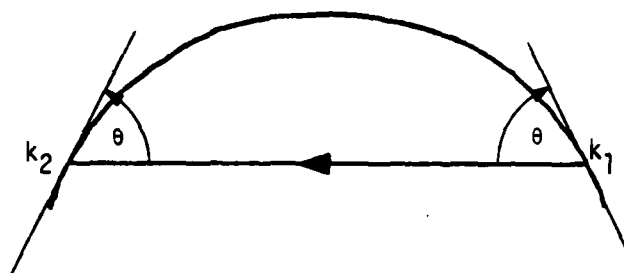
but $\qquad k_1 > \frac{\sin \theta}{c} \Rightarrow \frac{\partial^2 (A/c^2)}{\partial s_1^2} = \frac{-1}{2c \cos \theta}(\frac{\sin \theta_1}{c} - k_1) > 0$
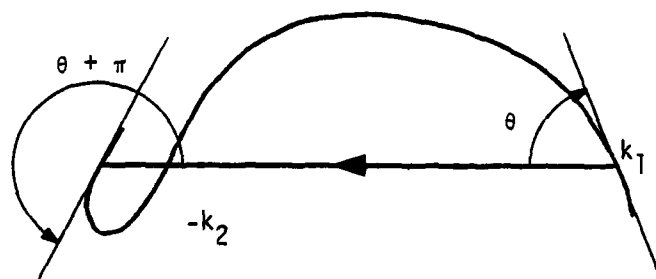
so we have a local minimum (by eq. 7).

If $\qquad k_1 k_2 - \frac{\sin \theta}{c}(k_1 + k_2) = 0$, then

$$k_1 = \frac{\sin \theta}{c} + \frac{\sin \theta}{c} \frac{k_1}{k_2} > \frac{\sin \theta}{c} \text{ and again } \frac{\partial^2 (A/c^2)}{\partial s_1^2} > 0$$

In this case we may or may not have a local minimum; we certainly do not have a maximum.

(a)

(b)

Figure 4.

3. Behavior of $^A/_C2$ on polygons

A special case of interest is that of polygonal shapes. Consider a chord connecting two corners of a polygon (Figure 5a). (Note that we can include the case of the chord ending on the side of a polygon by considering a "corner" with a 180° angle.) In Figure 5b, the origin represents the position of the chord in 5a, and the horizontal and vertical axes represent displacements from P_1 and P_2, respectively. From this figure we can see that the values of $^A/_C2$ are given by four different functions, one for each quadrant. To see if the origin is a local extreme value, we can find an expression for $\Delta(^A/_C2)$ for each of the four quadrants, and examine its sign. Let Δ_i be a displacement in the direction of θ_i. (Figure 5c) Let $\Delta_{ij}(^A/_C2)$ be the change in $^A/_C2$ resulting from the displacement (Δ_i, Δ_j). We can then examine the sign of Δ_{ij} for $(i,j) \in \{(1,2),(3,2),(3,4),(1,4)\}$ as Δ_i and $\Delta_j \to 0$. If the sign is always positive, the chord yields a local minimum, if the sign is always negative we have a maximum.

In the case of a polygon, we can derive an exact expression for $\Delta_{ij}(^A/_C2)$ (Figure 6). The equations in Figure 6 can be simplified to yield

$$\Delta_{ij}(A) = -\frac{1}{2}C(\Delta_i \sin \theta_i + \Delta_j \sin \theta_j) + \frac{1}{2} \Delta_i\Delta_j \sin(\theta_i + \theta_j)$$

$$\Delta_{ij}(C^2) = \Delta_i^2 + \Delta_j^2 - 2C(\Delta_i \cos \theta_i + \Delta_j \cos \theta_j) + 2\Delta_i\Delta_j \cos(\theta_i + \theta_j)$$

We wish to find

$$\Delta_{ij}(^A/_C2) = \frac{A+\Delta_{ij}(A)}{C^2+\Delta_{ij}(C^2)} - \frac{A}{C^2} = \frac{\Delta_{ij}(A)C^2 - A\cdot\Delta_{ij}(C^2)}{C^2(C^2+\Delta_{ij}(C^2))}$$

Since the denominator will be positive as $\Delta_i, \Delta_j \to 0$, we can restrict our attention to the sign of the numerator

$$\Delta_{ij}(A)\cdot C^2 - \Delta_{ij}(C^2)\cdot A$$

(a)

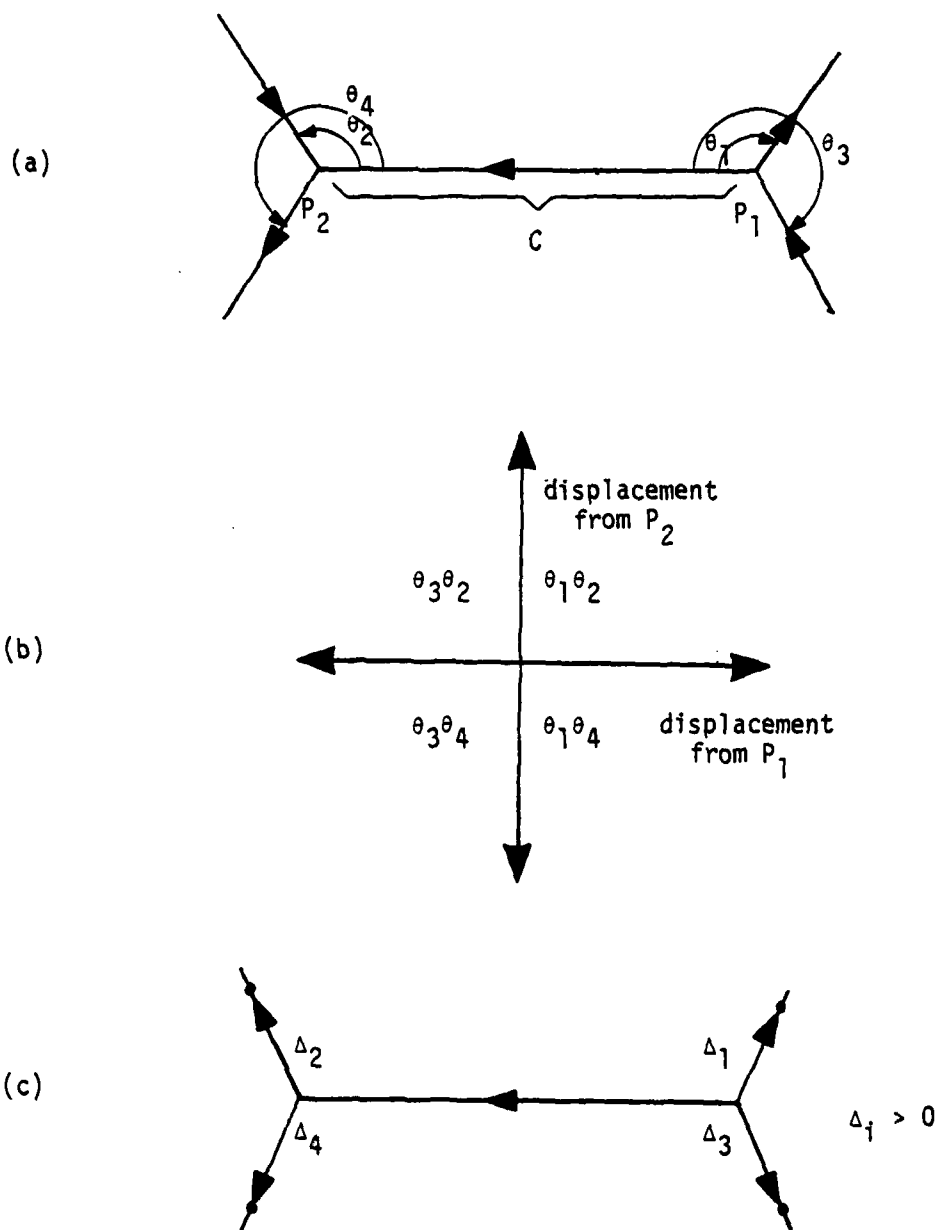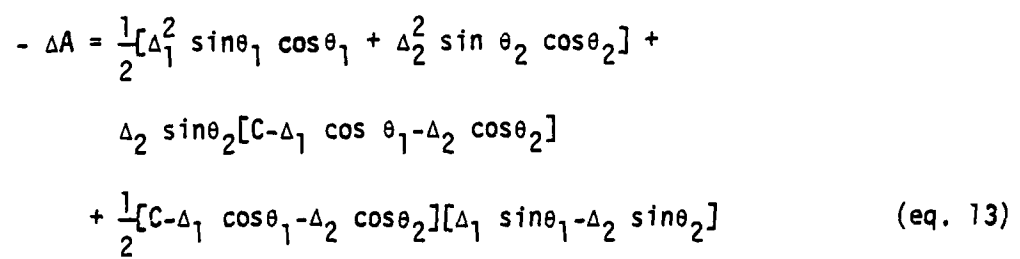θ_4
θ_2
θ_1
θ_3

P_2

P_1

C

(b)

displacement
from P_2

$\theta_3\theta_2$ $\theta_1\theta_2$

displacement
from P_1

$\theta_3\theta_4$ $\theta_1\theta_4$

(c)

Δ_2

Δ_1

Δ_4

Δ_3

$\Delta_i > 0$

Figure 5.

$$-\Delta A = \frac{1}{2}[\Delta_1^2 \sin\theta_1 \cos\theta_1 + \Delta_2^2 \sin\theta_2 \cos\theta_2] +$$

$$\Delta_2 \sin\theta_2[C-\Delta_1 \cos\theta_1-\Delta_2 \cos\theta_2]$$

$$+ \frac{1}{2}[C-\Delta_1 \cos\theta_1-\Delta_2 \cos\theta_2][\Delta_1 \sin\theta_1-\Delta_2 \sin\theta_2] \qquad \text{(eq. 13)}$$

$$(C+\Delta C)^2 = [C-\Delta_1 \cos\theta_1-\Delta_2 \cos\theta_2]^2 + [\Delta_1 \sin\theta_1-\Delta_2 \sin\theta_2]^2 \qquad \text{(eq. 14)}$$

Figure 6.

To first order in Δ_i and Δ_j, this is

$$\Delta_i[2AC \cos \theta_i - \frac{C^3}{2} \sin \theta_i] + \Delta_j[2AC \cos \theta_j - \frac{C^3}{2} \sin \theta_j]$$

We will clearly have a local maximum if both coefficients of the deltas are negative:

$$\frac{4A}{C^2} \cos \theta_i < \sin \theta_i \text{ and } \frac{4A}{C^2} \cos \theta_j < \sin \theta_j$$

The geometric consequences of this are illustrated in Figure 7, along with the corresponding case for a local minimum. The coefficients of Δ_i and Δ_j will be negative iff the sides of the polygon lie in the shaded region, as illustrated.

If either coefficient is positive, we have no local maximum. The only remaining cases are when one coefficient is zero and the other negative, or when both are zero. Suppose the coefficient of Δ_i is zero, and that of Δ_j is negative. If we consider a displacement from the origin (Figure 5b) in any direction for which Δ_j is non-zero, then clearly as the magnitude of the displacement goes to zero the first order terms will dominate, and $\Delta(^A/_C2)$ will go negative. If we consider a displacement for which $\Delta_j = 0$, then the first order terms of $\Delta_{ij}(^A/_C2)$ are all zero, and of the second order terms, which are

$$-A(\Delta_i^2 + \Delta_j^2) + \Delta_i\Delta_j[\frac{1}{2}C^2 \sin(\theta_i + \theta_j) -2A \cos(\theta_i + \theta_j)]$$

only the term $-A \Delta_i^2$ remains, thus $\Delta_{ij}(^A/_C2) < 0$. (Remember we want local maxima for which $A > 0$.) Finally, if both coefficients are zero, we have

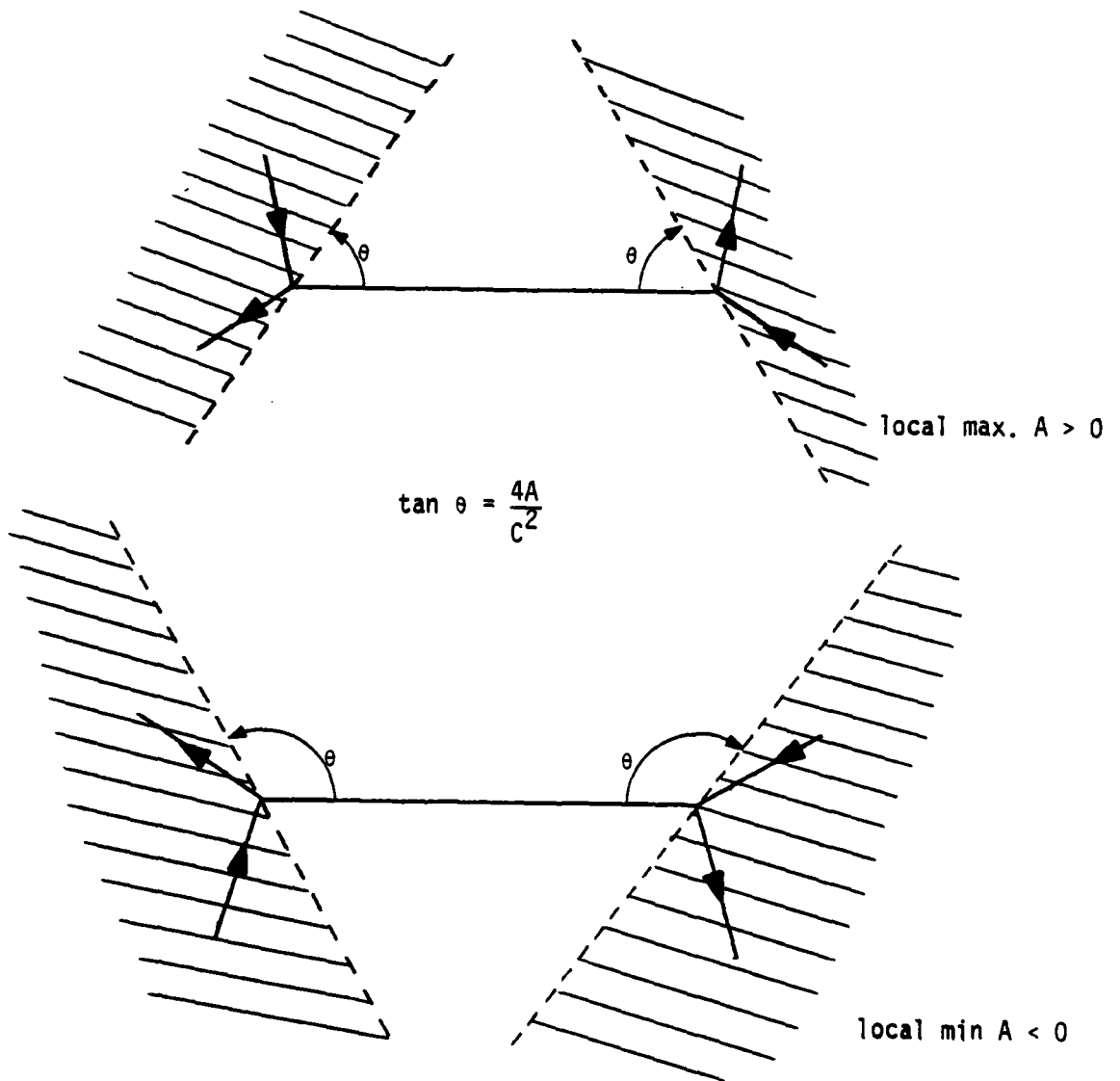$$\tan \theta_i = \frac{4A}{C^2} \text{ and } \tan \theta_j = \frac{4A}{C^2}$$

$$\tan \theta = \frac{4A}{C^2}$$

local max. A > 0

local min A < 0

Figure 7.

There are then two possibilities. If $\theta_i = \theta_j + \pi$ then we find

$$\Delta_{ij}(^A/_C2) = -A(\Delta_i^2 + \Delta_j^2) + \Delta_i\Delta_j[\tfrac{1}{2}C^2 \sin(2\theta_i + \pi) - 2A\cos(2\theta_i + \pi)]$$

and since $\sin(2\theta_i + \pi) = -\sin(2\theta_i) = -2\sin\theta_i\,\cos\theta_i = -\dfrac{8A}{C^2}\cos^2\theta_i$

$$\cos(2\theta_i + \pi) = -\cos(2\theta_i) = 1 - 2\cos^2\theta_i$$

we find

$$\Delta_{ij}(^A/_C2) = -A(\Delta_i^2 + \Delta_j^2) + \Delta_i\Delta_j[-4A\cos^2\theta_i - 2A + 4A\cos^2\theta_i]$$

$$\therefore \quad \Delta_{ij}(^A/_C2) = -A(\Delta_i^2 + \Delta_j^2) - 2A\,\Delta_i\Delta_j = -A(\Delta_i + \Delta_j)^2 < 0$$

If $\theta_i = \theta_j$ we find $\Delta_{ij}(^A/_C2) = -A(\Delta_i^2 + \Delta_j^2) + 2A\,\Delta_i\Delta_j$. Since this yields $\Delta_{ij}(^A/_C2) = 0$ for $\Delta_i = \Delta_j$, we have no maximum. This result is intuitively expected, as in this case C changes proportionately to Δ_i and Δ_j, and A changes proportionately to $\Delta_i^2 = \Delta_j^2$.

In summary, we will have no positive local maximum or negative minimum iff:

i) any of the θ_k, $k = 1,2,3,4$ lie outside the range depicted in Figure 7, or

ii) there is a pair $(\theta_i,\theta_j),(i,j) \in \{(1,2),(3,2),(3,4),(1,4)\}$ such that $\theta_i = \theta_j = \arctan\left(\dfrac{4A}{C^2}\right)$. In Figure 7 this would correspond to two sides touching opposite ends of the chord, both of which lie along the dashed lines, and both of which lie on the same side of the chord.

Observe that the above implies that we can never have a local maximum at a chord for which both endpoints lie on sides of the polygon, but that it is possible to have a maximum at a chord with one endpoint on a corner and the other endpoint on a side, if the side lies along the dashed line in Figure 7, and neither angle of the corner is in the direction of the dashed line.

4. Other measures

a) A/C: Setting the first partial derivatives equal to zero for this function yields $\tan \theta_i = \dfrac{2A}{C^2}$, $i = 1,2$. Applying the test involving second derivatives (eq. 7) leads to a cumbersome expression.

The behavior of this function on a polygon is similar to the behavior of A/C^2 illustrated in Figure 7, except that the slope of the dashed lines is given by $\dfrac{2A}{C^2}$ rather than $\dfrac{4A}{C^2}$. The case in which the sides of the polygon lie along the dashed lines has not been analyzed for A/C.

b) S/C, where S = arc length of the segment of curve cut off by the chord.* Setting the first derivatives equal to zero and using $\dfrac{\partial S}{\partial s_i} = -1$ (see Figure 1), we obtain $\cos \theta_i = C/S$, $i = 1,2$.

For the case of a polygon, we can carry out an analysis analagous to that for A/C^2. Referring to Figure 5, observe that movement along Δ_1 or Δ_2 decreases S, while movement along Δ_3 or Δ_4 increases S. Thus we must consider several cases. Also, since ΔC would involve square roots, we will consider $\Delta(S^2/C^2)$.

$$\Delta_{12}\left(\frac{S^2}{C^2}\right) = \frac{\Delta_{12}(S^2)C^2 - S^2\Delta_{12}(C^2)}{C^2(C^2 + \Delta_{12}(C^2))}$$ so we are interested in the sign of

$$\Delta_{12}(S^2)C^2 - S^2\Delta_{12}(C^2) \qquad\qquad (eq.\ 13)$$

$$\Delta_{12}(S^2) = (S + \Delta S)^2 - S^2 = 2S\,\Delta S + (\Delta S)^2 = 2S(-\Delta_1 - \Delta_2) + (\Delta_1 + \Delta_2)^2$$

We find (considering the first order terms) that we will have a local maximum if

$$\cos \theta_i < C/S \text{ for } i = 1,2$$

---

*Since this measure is never smaller than 1, we are interested only in local maxima.

By similar reasoning, we find $\cos \theta_i < -\frac{C}{S}$ for $i = 3,4$. This is illustrated geometrically in Figure 8.

The second order terms for (eq. 13) are

$$(\Delta_1 + \Delta_2)^2 C^2 - S^2 (\Delta_1^2 + \Delta_2^2 + 2\Delta_1\Delta_2 \cos [\theta_1 + \theta_2]).$$

Considering the various cases we find that we have a maximum except in the case when θ_i and θ_j lie on the dashed lines on the same side of the chord, in which case $\Delta_{ij}(S/C) = 0$ for $\Delta_i = \Delta_j$, $(i,j) \in \{(1,2),(3,2),(3,4),(1,4)\}$. This is the result we would anticipate intuitively. When $(i,j) \in \{(1,2), (3,4)\}$ both C and S change proportionately to $\Delta_i = \Delta_j$. For $(i,j) \in \{(3,2),(1,4)\}$ both C and S remain constant if $\Delta_i = \Delta_j$. (See Figure 9.)

c) $A/S2$: Setting the first partials to zero yields $\sin \theta_i = \frac{4A}{SC}$. Further analysis is not given due to the cumbersome expressions and the poor experimental results.

d) Finding negative minima of A to locate intrusions: The conditions under which we will have a negative minimum for A are quite simple, and can be expressed for any shape, smooth curve or polygon, by the statement that the curve in the immediate vicinity of the chord endpoints must lie to the left of the chord. Referring to Figure 1, if we take a displacement Δs_i from the point defined by s_i $(i=1,2)$, then for Δs_i small enough, the point $s_i + \Delta s_i$ must lie to the left of the directed chord. Several such cases are illustrated in Figure 10.

$$\cos \theta_i < \frac{C}{S} \quad i = 1,2$$

$$\cos \theta = \frac{C}{S}$$

$$\cos \theta_i < -\frac{C}{S} \quad i = 3,4$$

$$\cos \theta = -\frac{C}{S}$$
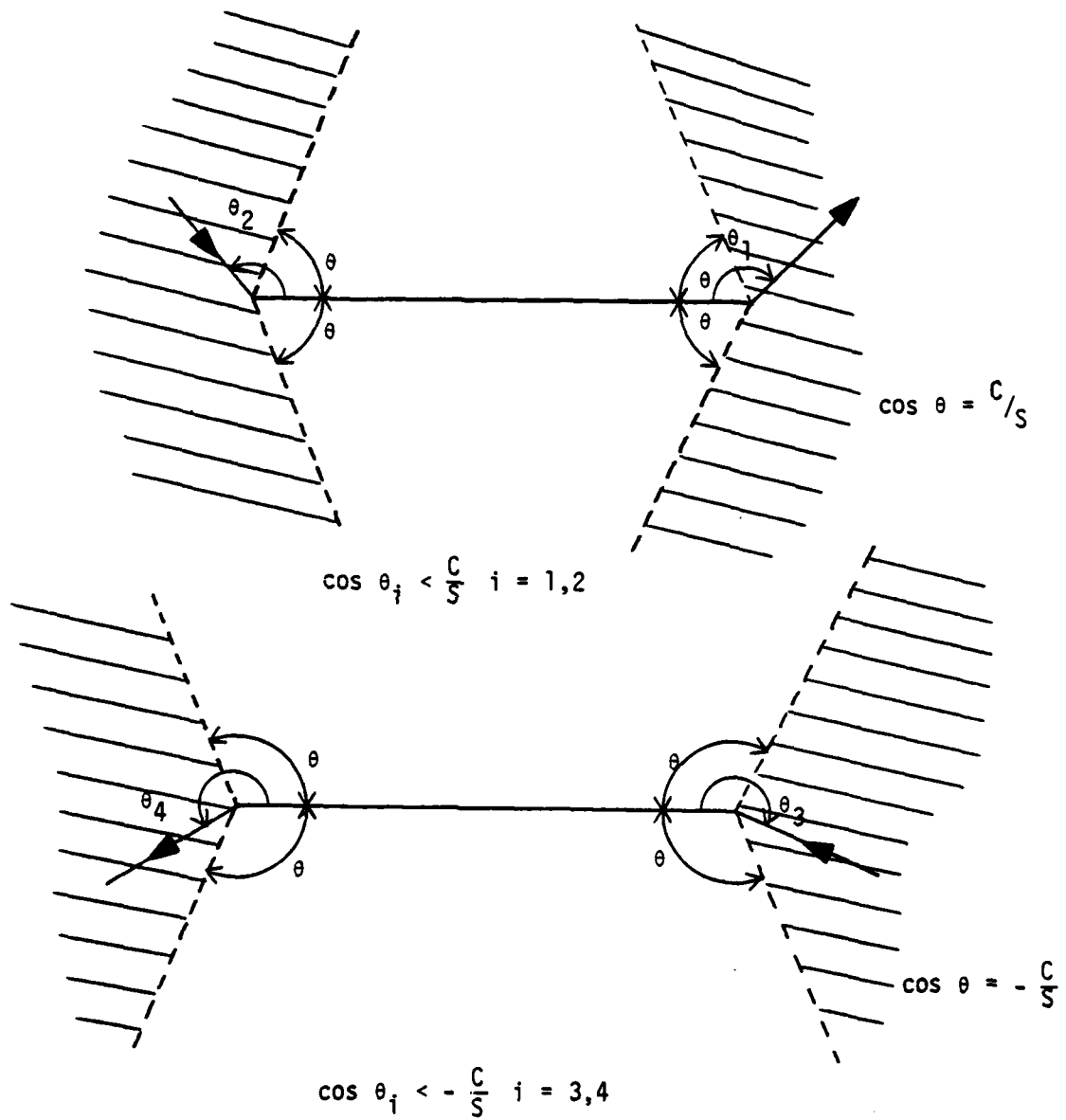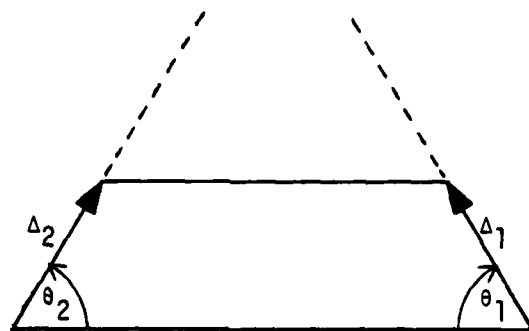
Figure 8.

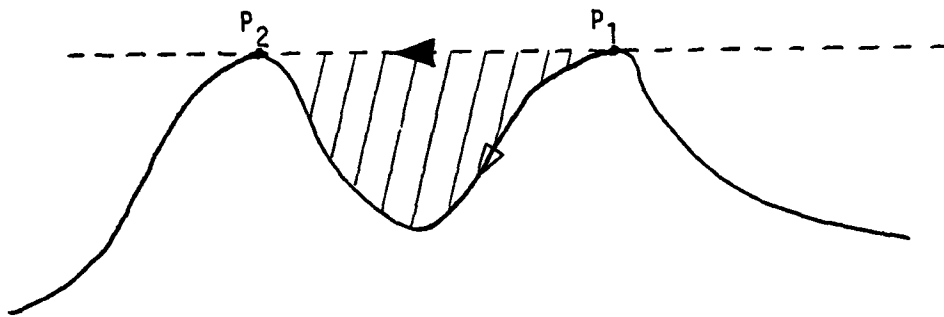$\theta_1 = \theta_2,\ \cos\theta_1 = \dfrac{C}{S}$

$\Delta_1 = \Delta_2$
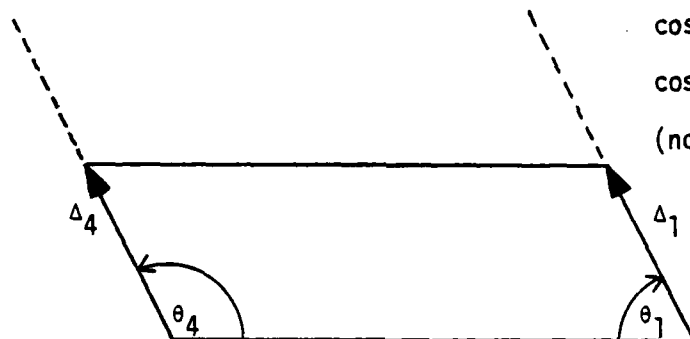
$\cos\theta_1 = \dfrac{C}{S}$

$\cos\theta_2 = -\dfrac{C}{S} \qquad \Delta_1 = \Delta_4$

(note, $\Delta_{14}S = -\Delta_1 + \Delta_4 = 0$)
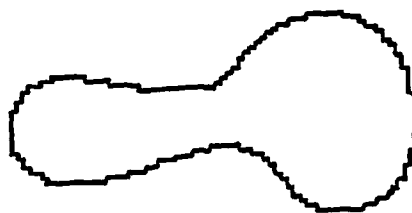
Figure 9.
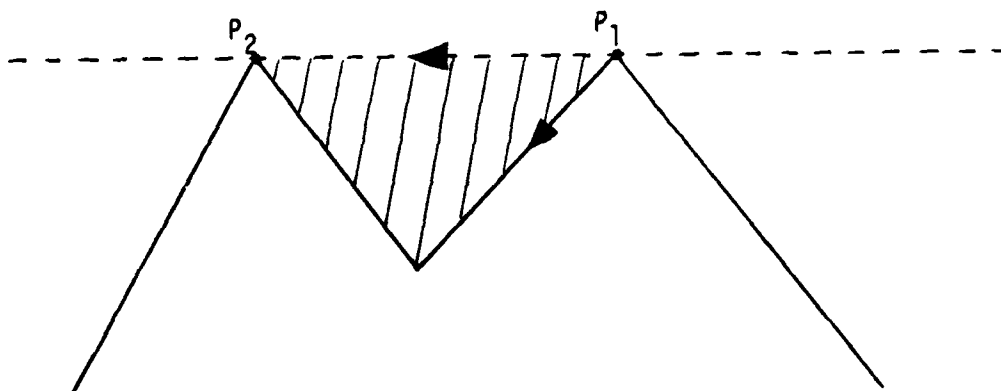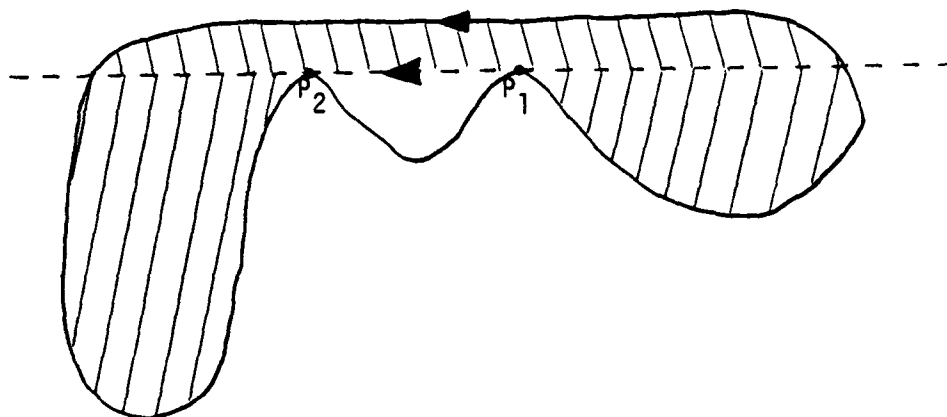
Shaded area is minimized.

Figure 10.

5. Conclusions

From the foregoing analysis we can anticipate some unusual and perceptually "unnatural" results. This is borne out by the experimental results on three shapes shown in Figure 11. In the experimental results, chords were suppressed if the segment they cut off exceeded a certain percentage of the total boundary perimeter, or if they cut through the object boundary.

For example, referring to Figure 6, we can see that a polygon with one side lying outside the indicated range will not be segmented, no matter how sharp the corners. (Observe, however, that given two points on the shape, there are two directed chords passing through these points, and although one chord may not be a segmentation chord, the other may; see Figure 12a.) Similarly, on a smooth curve, a protrusion or an intrusion may not be detected, no matter how perceptually obvious, because there is no chord for which θ_1 and θ_2 meet the necessary conditions. For example, we can observe this effect in the experimental results on shape 2 in Figure 11. Here, the slopes on the sides of the concavity did not meet the criterion $\tan \theta = \frac{4A}{c^2}$ for the measure A/c^2, but did meet the criterion $\tan \theta = \frac{2A}{c^2}$ for A/c. Thus we can see that we can, for example, have a large convex lobe which will never be detected by A/c^2 because the slope of the sides never equals $4A/c^2$ (Figure 12b).

Consider the situation which occurs on part of the E shape (Figure 12c). For the chord $\overline{P_1P_2}$ to be a local maximum of A/c^2, we must have $\tan \theta = 4A/c^2$. For this piece of shape, $\tan \theta = \frac{W}{x}$, $A = LW + \frac{1}{2}xW$ and $c^2 = W^2 + x^2$ so we find we must have $x = -2L \pm \sqrt{4L^2+W^2}$, and since we cannot have $x < L$, $x = \sqrt{4L^2+W^2} - 2L$. A similar analysis for A/c yields $x = \frac{W^2}{2L}$. Therefore,
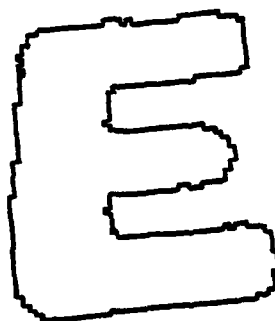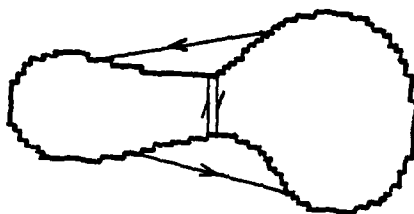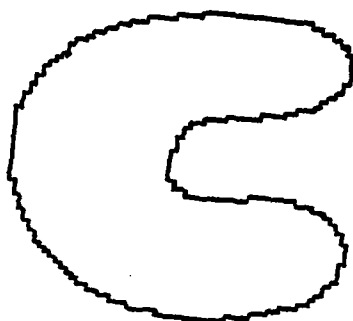
(1)

(2)
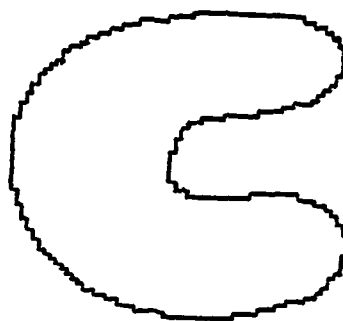
(3)

Figure 11.   Results of applying the methods described in
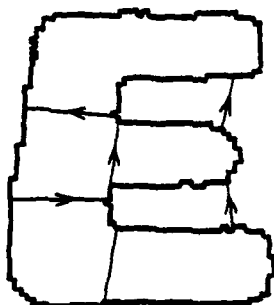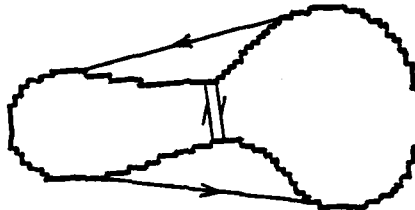this paper to three shapes

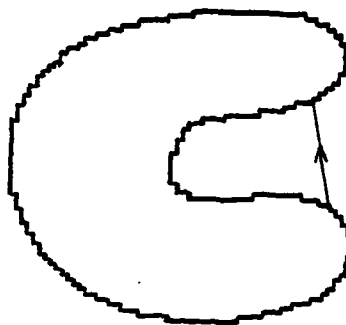(a) The shapes

(1)

(2)

(3)

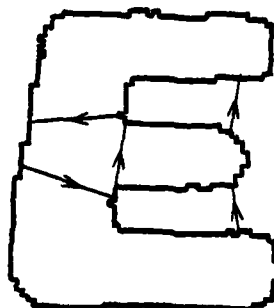Figure 11, continued: (b) Maxima of A/C^2
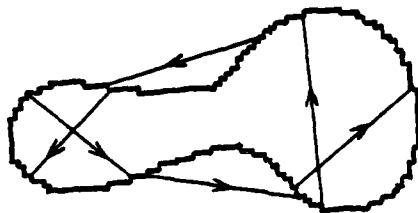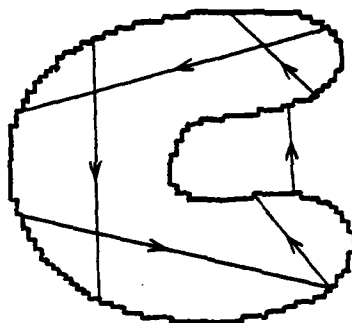
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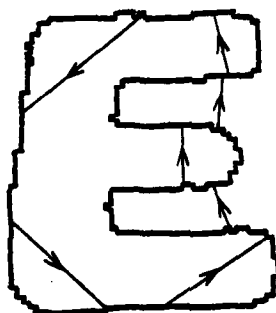
Figure 11, continued: (c) Maxima of A/C
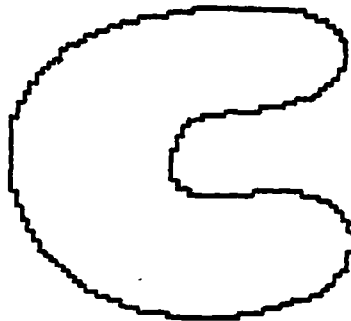
(1)

(2)

(3)

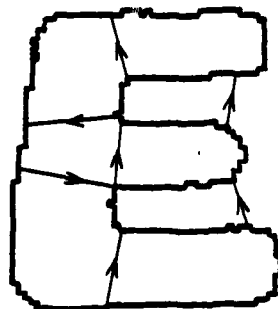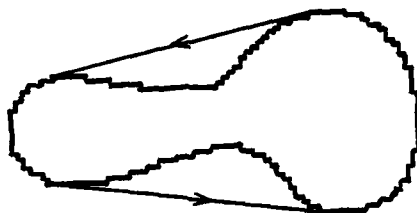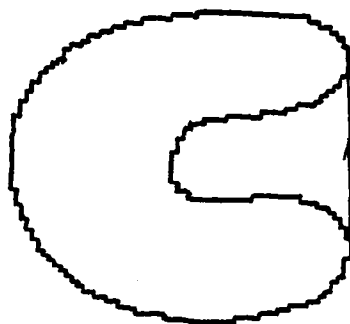Figure 11, continued: (d) Maxima of A/S^2

(1)

(2)

(3)

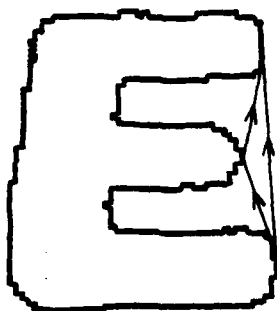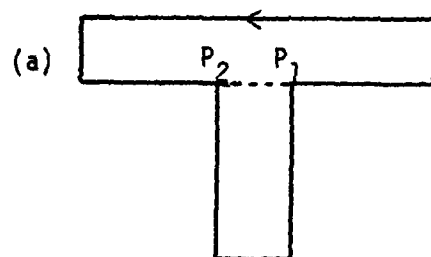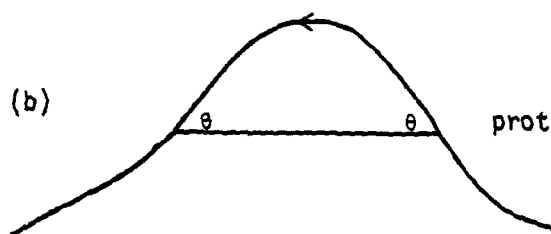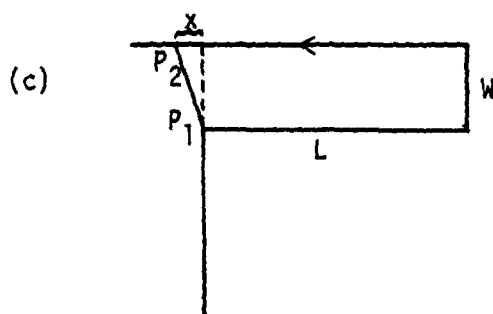Figure 11, continued: (e) Maxima of S/C

(1)

(2)

(3)

Figure 11, continued: (f) Negative maxima of A

(a) Chord $\overline{P_1P_2}$ will not segment the object, but chord $\overline{P_2P_1}$ will.

(b) protrusion for which $\tan \theta < \dfrac{4A}{C^2}$

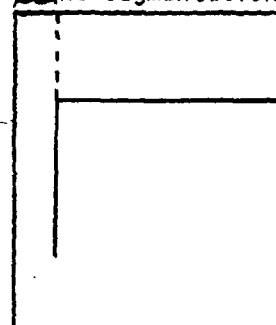(c) if this distance is too small, there is no segmentation here.

Figure 12.

whether or not the tip of the E is segmented by chord $\overline{P_1 P_2}$ depends on the
location of the upper left corner (Figure 12c). Again we have a perceptually
obvious feature whose segmentation depends in a critical way on a dimension
which seems to have little perceptual significance. (In fact, narrowing the
vertical portion of the E would tend to make the horizontal protrusions
even more obvious.)

In summary, the principal flaw with these measures is that whether or
not segmentation is achieved depends on certain features that have little
perceptual significance. Thus we can have a variety of shapes which appear
to a human observer virtually identical, and yet which yield very different
results in segmentation. Of course, any method of segmentation will have a
certain cutoff, or threshold in its behavior. However, we expect to find
such thresholds occurring in cases which represent a transition from one
perceptual situation to another (such as the transition from corner to
straight line as the angle of the corner approaches π). In many situations,
the thresholds for these methods do not correspond to any such perceptually
significant transition. Furthermore, we should note that these techniques
are also subject to problems with noise and digitization, just as is the
case with more conventional techniques. By using a global measure we do not
avoid the need for smoothing operations or other methods for locating noisy
peaks. For example, the maximum associated with the part of an E shape il-
lustrated in Figure 12c tends to be very gentle, and is easily obscured by
noise. In some cases the programs failed to locate this segmentation chord
for this reason.

Reference

1. A. Rosenfeld and A. C. Kak, Digital Picture Processing, Academic Press, N.Y., 1976, Section 9.3.2.

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This report investigates a class of shape segmentation methods in which, for each arc of the shape's contour, we consider the region bounded by the arc and its chord; compute a simple geometrical property of this region; and choose arcs for which this property's value is a local extremum. The characteristics of this approach are analyzed for several such properties, and examples of the segmentations obtained in this way are given. It is concluded that such methods will sometimes yield results that are not perceptually plausible.

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